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Vanishing scalar invariant spacetimes in supergravity

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Abstract: We show that the higher-dimensional vanishing scalar invariant (VSI) spacetimes with fluxes and dilaton are solutions of type IIB supergravity, and we argue that they are exact solutions in string theory. We also discuss the supersymmetry properties of VSI spacetimes.

Keywords: Supergravity Models, Penrose limit and pp-wave background.

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1. Introduction

VSI spacetimes are N-dimensional Lorentzian spacetimes in which all curvature invariants of all orders vanish [1]. Recently, we presented all of the metrics for the higher-dimensional VSI spacetimes, which can be of Ricci type N or III [2]. The Ricci type N VSI spacetimes include the higher-dimensional (generalized) pp-wave spacetimes, which have been the most studied in the literature and are known to be exact solutions of supergravity and in string theory. However, many of the mathematical properties of VSI spacetimes (in general, and the pp-wave spacetimes in particular) may not be familiar.

In this paper we will show that all Ricci type N VSI spacetimes are solutions of supergravity (and argue that Ricci type III VSI spacetimes are also supergravity solutions if supported by appropriate sources). The VSI Ricci type III supergravity solutions to be presented are new. We also find some new Ricci type N supergravity solutions. We explicitly study type IIB supergravity, but similar results are expected in all supergravity theories. We also argue that, in general, the VSI spacetimes are exact string solutions to all orders in the string tension.

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We then discuss which VSI supergravity spacetimes can admit supersymmetry. It is known that in general if a spacetime admits a Killing spinor, it necessarily admits a null or timelike Killing vector. Therefore, a necessary (but not sufficient) condition for a particular supergravity solution to preserve some supersymmetry is that the spacetime possess such a Killing vector. In the appendix we prove that VSI spacetimes whose metric functions have dependence on the light-cone coordinate v cannot possess a null or timelike Killing vector. Hence, only VSI spacetimes with a covariantly constant null vector are candidates to preserve supersymmetry. Such spacetimes include not only pp-waves but also (the more general) spacetimes of algebraic Weyl type III(a).

We therefore study the supersymmetry properties of VSI type IIB supergravity solutions with a covariantly constant null vector. We focus on solutions of Weyl type III(a), since type N spacetimes have been studied extensively. We conclude there are no such supersymmetric solutions in the vacuum type III(a) case. We present explicit examples of Weyl type III(a) NS-NS (one-half) supersymmetric solutions.

1.1 Higher-dimensional VSI metrics

All curvature invariants of all orders vanish in an N-dimensional Lorentzian (VSI) spacetime if and only if there exists an aligned shear-free, non-expanding, non-twisting, geodesic null direction ℓ^a along which the Riemann tensor has negative boost order [1]. The VSI spacetimes can be classified according to their Weyl type, Ricci type and the vanishing or non-vanishing of the 'spin coefficient' ε [3]. In [2], the explicit metric forms for higher dimensional VSI spacetimes were presented.

Since VSI spacetimes possess a null vector field ℓ obeying

$$\ell^A \ell_{B;A} = \ell^A_{\ ;A} = \ell^{A;B}\ell_{(A;B)} = \ell^{A;B}\ell_{[A;B]} = 0; \tag{1.1}$$

i.e., ℓ is geodesic, non-expanding, shear-free and non-twisting, the VSI spacetimes belong to the higher-dimensional *Kundt* class [1]. It follows that any VSI metric can be written in the form

$$\mathrm{d}s^2 = 2\mathrm{d}u\left[\mathrm{d}v + H(v,u,x^n)\mathrm{d}u + W_i(v,u,x^n)\mathrm{d}x^i\right] + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{1.2}$$

with $i,j = 1,\ldots,N-2$. The negative boost order conditions of the Riemann tensor yield [2]

$$W_i(v,u,x^k) = vW_i^{(1)}(u,x^k) + W_i^{(0)}(u,x^k), \tag{1.3}$$

$$H(v,u,x^k) = \frac{v^2}{8}(W_i^{(1)})(W^{(1)i}) + vH^{(1)}(u,x^k) + H^{(0)}(u,x^k). \tag{1.4}$$

The $W_i^{(1)}$ are subject to further differential constraints: using the allowable freedom we can choose

$$W_1^{(1)} = -2\frac{\varepsilon}{x^1}; \quad W_n^{(1)} = 0, \ \ n = 2,\ldots,N-2 \tag{1.5}$$

(where $\varepsilon = 0$ corresponds to $W_1^{(1)} = 0$ and $\varepsilon = 1$ corresponds to $W_1^{(1)} \neq 0$). Accordingly,

$$H(v,u,x^k) = \frac{v^2\varepsilon}{2(x^1)^2} + vH^{(1)}(u,x^k) + H^{(0)}(u,x^k). \tag{1.6}$$

We note that, in general, for the Kundt metrics there exist coordinate transformations $x'^j = f^j(u, x^i)$ which can be used to simplify *either* the transverse metric *or* the functions $W_i^{(0)}(u, x^k)$. Here we have used this to eliminate the u-dependence in the transverse metric to get the explicitly flat metric $\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$ (and hence in these coordinates the W_i are not zero). Under the remaining allowable coordinate transformations we obtain $H^{(0)}(u, x^k) \to H^{(0)}(u, x^k) - (h(u, x^k))_{,u}$, so that we can redefine $W_i^{(0)}$ and essentially freely specify $H^{(0)}$ (e.g., we could set $H^{(0)}$ to zero), and in the case $\varepsilon = 0$, we obtain $H^{(1)}(u, x^k) \to H^{(1)}(u, x^k) + G(u)$ (after redefining $H^{(0)}$ and $W_i^{(0)}$), where $G(u)$ is freely specifiable.

All of these spacetimes are VSI. The spacetimes above are in general of Ricci and Weyl type III. Further progress can be made by classifing the metric in terms of their Weyl type (III, N or O) and their Ricci type (N or O) [3], and the form of ε. The metric functions H and W_i satisfy the remaining Einstein equations. In table 1 in [2], all of the VSI spacetimes supported by appropriate bosonic fields are presented and the metric functions are listed.[1] It is the higher-dimensional (generalized) pp-wave spacetimes that have been most studied in the literature. It is known that such spacetimes are exact solutions in string theory [4 – 6], in type-IIB superstrings with an R-R five-form [7], also with NS-NS form fields [8]. In higher dimensions, VSI supergravity solutions can be constructed [5], and we shall see that all VSI spacetimes are solutions of superstring theory when supported by appropriate bosonic fields.

It is convenient to introduce the null frame

$$\boldsymbol{\ell} = \mathrm{d}u, \tag{1.7}$$

$$\mathbf{n} = \mathrm{d}v + H\mathrm{d}u + W_i\mathbf{m}^{i+1}, \tag{1.8}$$

$$\mathbf{m}^{i+1} = \mathrm{d}x^i. \tag{1.9}$$

The Weyl tensor can then be expressed as [3]

$$C_{abcd} = 8\Psi_i\ell_{\{a}n_b\ell_c m^i_{d\}} + 8\Psi_{ijk}m^i_{\{a}m^j_b\ell_c m^k_{d\}} + 8\Psi_{ij}\ell_{\{a}m^i_b\ell_c m^j_{d\}}. \tag{1.10}$$

The case $\Psi_{ijk} \neq 0$ is of Weyl type III, while $\Psi_{ijk} = 0$ (and $\Psi_i \equiv 2\Psi_{ijj} = 0$) corresponds to type N. Further subclasses of type III can be considered; for example, type III(a) where $\Psi_i = 0$ but $\Psi_{ijk} \neq 0$. The Ricci tensor is given by

$$R_{ab} = \Phi\ell_a\ell_b + \Phi_i(\ell_a m^i_b + \ell_b m^i_a). \tag{1.11}$$

The Ricci type is N if $\Phi_i = 0 = R_{1i}$ (otherwise the Ricci type is III; Ricci type O is vacuum). When $\varepsilon = 0, 1$ the Ricci type N conditions $\Phi_i = 0$ reduce to

$$2H^{(1)}{}_{,1} = \frac{2\varepsilon}{x^1}W^{(0)m}{}_{,m} - W^{(0)m}{}_{,m1} \tag{1.12}$$

$$2H^{(1)}{}_{,n} = \Delta W_n^{(0)} - W^{(0)m}{}_{,mn} \tag{1.13}$$

[1] The Christoffel symbols needed for the calculations are: In $\varepsilon = 0$ VSI spacetimes, $\Gamma_{\lambda v}{}^{\lambda} = \Gamma_{\lambda i}{}^{\lambda} = 0$ for any choice of λ. In $\varepsilon = 1$ VSI spacetimes, $\Gamma_{\lambda v}{}^{\lambda} = 0$, $\Gamma_{u1}{}^{u} = -\Gamma_{v1}{}^{v} = -\frac{1}{2}W_{1,v}$.

subject to

$$\Delta W^{(0)}_{n,1} = \frac{2\varepsilon}{x^1} W^{(0)m}{}_{,mn}, \quad \Delta W^{(0)}_{m,n} = \Delta W^{(0)}_{n,m}, \tag{1.14}$$

where $\Delta = \partial^i \partial_i$ is the spatial Laplacian and $m, n \geq 2$. As a result, in Ricci type N spacetimes $H^{(1)}$ can be determined as a function of the $W^{(0)}_i$ (in contrast to the Ricci type III case).

For the VSI spacetimes, the aligned, repeated, null vector ℓ is a null Killing vector (KV) if $\ell_{1;1} = 0 = \ell_{(1;i)}$ (i.e., $\varepsilon = 0$), whence $H_{,v} = 0$ and $W_{i,v} = 0$ and the metric no longer has any v dependence. Furthermore, since $L_{AB} := \ell_{A;B} = \ell_{(A;B)}$ it follows that in this case if ℓ is a null KV then it is also covariantly constant. In general, the higher-dimensional VSI metrics admitting a null KV (and hence a covariantly constant null vector (CCNV)) are of Ricci and Weyl type III [2]. The subclass of Ricci type N CCNV spacetimes are related to the ($F = 1$) chiral null models of [5]. The subclass of Ricci type N and Weyl type III(a) spacetimes includes the relativistic gyratons [9]. The subclass of Ricci type N and Weyl type N spacetimes are the generalized pp-wave spacetimes.

2. VSI spacetimes in IIB supergravity

Our aim is to construct bosonic solutions of IIB supergravity [14] for which the spacetime is VSI. We consider solutions with non-zero dilaton, Kalb-Ramond field and RR 5-form. The corresponding field equations[2] are

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -2\nabla_\mu \partial_\nu \phi + \frac{1}{4} H_{\mu\lambda\rho} H_\nu{}^{\lambda\rho} + e^{2\phi} \frac{1}{4 \cdot 4!} F_{\mu\lambda\rho\kappa\sigma} F_\nu{}^{\lambda\rho\kappa\sigma} \tag{2.1}$$

$$0 = F_{ijklm} H^{klm} \tag{2.2}$$

$$\nabla_\mu \partial^\mu \phi = -\frac{1}{4} R + \frac{1}{4 \cdot 2 \cdot 3!} H^2 + \partial_k \phi \partial^k \phi \tag{2.3}$$

$$0 = \nabla_\lambda H^{\lambda\mu\nu} - 2(\partial_\lambda \phi) H^{\lambda\mu\nu} \tag{2.4}$$

$$H = dB \tag{2.5}$$

$$dF = 0 \tag{2.6}$$

$$F = *F \tag{2.7}$$

The VSI requirement implies that all curvature tensors must be of negative boost order. It is therefore reasonable that we similarly require that the quadratic terms in H and F in eq. (2.1) are also of negative boost order. Since H and F are forms (hence, antisymmetric), we must have $F = (F)_{-1} + (F)_0 + (F)_1$ (similarly for H), where $(\ )_b$ means projection onto the boost-weight b components. As an example, consider $(F)_1$ which can be written $(F_{\mu\lambda\rho\kappa\sigma})_1 = n_{[\mu} \xi_{\lambda\rho\kappa\sigma]}$ where $\xi_{\lambda\rho\kappa\sigma}$ is a four-form and $\xi_{\lambda\rho\kappa\sigma} n^\lambda = \xi_{\lambda\rho\kappa\sigma} \ell^\lambda = 0$. We then have the boost-weight 2 component (analogously for H):

$$\left(F_{\mu\lambda\rho\kappa\sigma} F_\nu{}^{\lambda\rho\kappa\sigma} \right)_2 = \frac{1}{25} n_\mu n_\nu \xi_{\lambda\rho\kappa\sigma} \xi^{\lambda\rho\kappa\sigma}.$$

[2]The equations of motion of IIB supergravity are given, for example, in [10].

The factor $\xi_{\lambda\rho\kappa\sigma}\xi^{\lambda\rho\kappa\sigma}$ is a sum of squares, so that requiring that boost-weight 2 components of the $H^2 + F^2$ terms should vanish thus implies that $\xi_{\lambda\rho\kappa\sigma} = 0$, and hence, $(F)_1 = 0$. A similar calculation of the boost-weight 0 components enables us to show that if the quadratic terms of eq. (2.1) only have negative boost weight terms then F and H only possess negative boost weight terms.[3] Assuming, in addition, that $\nabla_\mu\phi$ has negative boost order, we thus have:

$$\nabla\phi = (\nabla\phi)_{-1}\,, \quad H = (H)_{-1}\,, \quad F = (F)_{-1}\,.$$

Note that this immediately implies that eqs. (2.2) and (2.3) are satisfied. Furthermore, this also means that the forms can be written

$$H_{\mu\nu\rho} = \ell_{[\mu}\tilde{B}_{\nu\rho]},\ F_{\mu\lambda\rho\kappa\sigma} = \ell_{[\mu}\varphi_{\lambda\rho\kappa\sigma]} \tag{2.8}$$

and $\ell^\mu\tilde{B}_{\mu\nu} = \ell^\mu\varphi_{\mu\rho\kappa\sigma} = 0$, $n^\mu\tilde{B}_{\mu\nu} = n^\mu\varphi_{\mu\rho\kappa\sigma} = 0$; i.e., $\tilde{B}$ and φ only have transverse components. By calculating $\mathrm{d}\phi$ and requiring this only possess negative boost order terms, we obtain $\phi = \phi(u)$.

2.1 Ricci type N solutions

We first construct solutions with Ricci type N VSI spacetimes. We postulate the following ansatz motivated by the preceding argument

$$g_{\mu\nu} = g^{\rm VSI}_{\mu\nu},\ \phi = \left\{ \begin{array}{ll} \phi(u) & (\varepsilon = 0) \\ \phi_0 & (\varepsilon = 1) \end{array} \right\},\ H_{\mu\nu\rho} = \frac{1}{4}\ell_{[\mu}\tilde{B}_{\nu\rho]},\ F_{\mu\lambda\rho\kappa\sigma} = \ell_{[\mu}\varphi_{\lambda\rho\kappa\sigma]} \tag{2.9}$$

where ϕ_0 is a constant, $\tilde{B}$ and φ are a two- and four-forms with no dependence on v, and ℓ is the null vector field in (1.1). Eqs. (2.2), (2.3) are automatically satisfied. From eqs. (2.5), (2.6), $\tilde{B}_{\nu\rho} = \tilde{B}_{\nu,\rho} - \tilde{B}_{\rho,\nu}$ [4] and φ has to be a closed form. We are left with the following equations

$$\begin{aligned} x^1\triangle\left(\frac{H^{(0)}}{x^1}\right) + \left(\frac{W^{(0)m}W^{(0)}_m}{x^1}\right)_{,1} - 2H^{(1)}{}_{,m}\,W^{(0)m} & \qquad (2.10) \\ -H^{(1)}W^{(0)m}{}_{,m} - \frac{1}{4}W_{mn}W^{mn} - W^{(0)m}{}_{,mu} &= -\frac{1}{4\cdot 4}\tilde{B}_{ij}\tilde{B}^{ij} - 3!\,e^{2\phi_0}\varphi^2 \quad (\varepsilon = 1) \\ \triangle H^{(0)} - \frac{1}{4}W_{mn}W^{mn} - 2H^{(1)}{}_{,m}\,W^{(0)m} & \qquad (2.11) \\ -H^{(1)}W^{(0)m}{}_{,m} - W^{(0)m}{}_{,mu} &= 2\phi'' + H^{(1)}\phi' \\ & \quad -\frac{1}{4\cdot 4}\tilde{B}_{ij}\tilde{B}^{ij} - 3!\,e^{2\phi}\varphi^2 \quad (\varepsilon = 0) \\ \partial_i\tilde{B}^{ij} &= 0 \qquad (2.12) \\ \varphi &= *_8\varphi \qquad (2.13) \end{aligned}$$

[3] One can, in principle, imagine a very special (and unnatural) situation where the derivatives of ϕ exactly cancel the non-negative boost-weight terms of H^2+F^2; however, we shall not consider this possibility further here.

[4] Note that the relationship between B, as defined in (2.5), and $\tilde{B}$ is $B_{\nu\rho} = \ell_{[\nu}\tilde{B}_{\rho]}$.

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In the above equations $m, n = 2, \ldots, 8$ and $i, j = 1, \ldots, 8$. Prime denotes derivative with respect to u and $*_8$ is the Hodge operator in the eight-dimensional transverse space.

The solutions above are of Weyl type III. They contain previously known solutions such as the string gyratons [11] and pp-wave supergravity solutions [7, 5, 6, 8]. The latter arise in the Weyl type N limit of the $\varepsilon = 0$ solutions (see [2]).

2.2 Ricci type III solutions

Ricci type III VSI spacetimes exist if appropriate source fields can be found. Recall that such a Ricci tensor must have boost weight -1 components. We note that for a general tensor product

$$(T \otimes S)_b = \sum_{b=b'+b''} (T)_{b'} \otimes (S)_{b''}$$

Hence, the quadratic terms in H and F in eq. (2.1) necessarily have boost weight -2. Projection of eq. (2.1) onto boost weight -1 components then gives

$$(R_{\mu\nu})_{-1} = -2\,(\nabla_\mu \partial_\nu \phi)_{-1}$$

We conclude that the space can only be of Ricci type III if $(\nabla_\mu \partial_\nu \phi)_{-1}$ is non-zero. This implies that $\varepsilon = 1$ and $\phi = \phi(u)$. Therefore, there are no Ricci III supergravity solutions where ℓ is a covariantly constant null vector (CCNV).[5]. This is perhaps unfortunate, as such solutions would have been good candidates to preserve supersymmetry.

Motivated by the preceding argument we construct a solution with non-constant dilaton $\phi = \phi(u)$ in the $\varepsilon = 1$ case. Eqs. (2.1) read

$$x^1 \triangle \left(\frac{H^{(0)}}{x^1}\right) + \left(\frac{W^{(0)m} W^{(0)}_m}{x^1}\right)_{,1} - 2H^{(1)}{}_{,m}\, W^{(0)m} \tag{2.14}$$

$$-H^{(1)} W^{(0)m}{}_{,m} - \frac{1}{4} W_{mn} W^{mn} - W^{(0)m}{}_{,mu} + \frac{2v}{(x^1)^2}\phi' = 2\phi'' + 2\left(H^{(1)} + \frac{v}{(x^1)^2}\right)\phi'$$

$$H^{(1)}{}_{,1} = \frac{1}{x^1} W^{(0)m}{}_{,m} - \frac{1}{2} W^{(0)m}{}_{,m1} + \frac{2}{x^1}\phi' \tag{2.15}$$

$$2H^{(1)}{}_{,n} = \Delta W^{(0)}_n - W^{(0)m}{}_{,mn} \tag{2.16}$$

Note that the v-dependent terms in (2.14) cancel each other. On the other hand, we can determine $H^{(1)}$ from (2.15), (2.16). The complete metric function is

$$H = H^{(0)}(u, x^i) + \frac{1}{2}\left(\tilde{F} - W^{(0)m}{}_{,m}\right) v + \frac{v^2}{2(x^1)^2}, \tag{2.17}$$

where $\tilde{F} = \tilde{F}(u, x^i)$ is a function satisfying

$$\tilde{F}_{,1} = \frac{2}{x^1}(W^{(0)m}{}_{,m} + 2\phi'), \quad \tilde{F}_{,n} = \Delta W^{(0)}_n. \tag{2.18}$$

[5]This is in the context of type IIB supergravity (with the sources considered) and the conditions described above. However, we cannot exclude the existence of supersymmetric Ricci type III solutions in more general situations.

The solution above is, to our knowledge, the first supergravity solution of Ricci type III. The dilaton dependence on u is crucial to construct the solution, and it reduces to Ricci type N when the dilaton is constant (or absent). VSI supergravity solutions of Ricci type III with form fields only do not exist. However, the solution above can be generalized in a straightforward way to include the form fields in (2.9).

The Ricci type N, Weyl type III solutions in the previous section can be reduced to Weyl type N. On the contrary, the Ricci type III solution presented here can only have Weyl type III.[6]

2.3 Solutions with non-zero $\mathbf{F_1}$, $\mathbf{F_3}$.

The above solutions can be generalized to include non-zero F_1, F_3 RR fields. It is well-known that $\mathrm{SL}(2,\mathbb{R})$ is the classical S-duality symmetry group for IIB supergravity (see, for example, [16]). Such a transformation can be parametrized by [21]

$$S = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \tag{2.19}$$

with $ps - qr = 1$. The fields transform according to

$$\begin{pmatrix} A_2' \\ B' \end{pmatrix} = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} A_2 \\ B \end{pmatrix}, \quad \tau' = \frac{p\tau + q}{r\tau + s} \tag{2.20}$$

where A_2 and B are the RR 2-potential and the Kalb-Ramond field, and $\tau = A_0 + ie^{-\phi}$, where A_0 is the RR scalar and ϕ is the dilaton. The metric and RR 5-form remain invariant.[7] The VSI solutions presented have $A_0 = A_2 = 0$. Under a transformation (2.19)

$$A_2' = qB, \quad B' = sB \tag{2.21}$$

In this way one can generate a non-zero F_3' which is proportional to H; the Kalb-Ramond field gets rescaled. The dilaton and RR scalar can be read from

$$sA_0' - e^{-(\phi+\phi')}r = q \tag{2.22}$$

$$rA_0' + se^{(\phi-\phi')}s = p \tag{2.23}$$

For solutions with $\phi = \phi(u)$ one obtains $(F_1')_u = \partial_u A_0'$. For solutions with a constant dilaton the RR 1-form remains zero.

2.4 String corrections

In four dimensions VSI spacetimes are known to be exact string solutions to all orders in the string tension α' even in the presence of additional fields [12]. Using the arguments of [4], higher-dimensional supergravity solutions supported by appropriate fields (e.g., with

[6] Requiring the solution to have vanishing boost weight -1 Weyl components reduces it to the Ricci and Weyl type N solution.

[7] The S-duality symmetry becomes manifest when the metric is in the Einstein frame: $\hat{g} = e^{\phi/2}g$. Such a rescaling does not affect the character of the VSI solutions presented.

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the dilaton and Kalb-Ramond field and appropriate form fields) are also known to be exact solutions in string theory [7, 5, 6, 8]. Similarly [12], it can be argued that the VSI supergravity spacetimes are exact string solutions to all orders in the string tension α', at least in the presence of a dilaton and Kalb-Ramond field. Higher-dimensional pp-waves are also exact solutions of string theory with RR fields (e.g., a F_5 field); it is to be expected, from an analysis of the perturbative counterterms, that this is also the case for the special VSI supergravity solutions under consideration here. Therefore, the VSI solutions presented may be of relevance in string theory. Note that these VSI spacetimes are, in general, time-dependent string theory backgrounds.

3. Supersymmetry

Given a spinor ϵ on a Lorentzian manifold, the vector constructed from its Dirac current

$$k^a = \bar{\epsilon}\gamma^a\epsilon \tag{3.1}$$

is null or timelike. Moreover, if ϵ is a Killing spinor then k^a is a Killing vector. This result has been proven for a number of supergravity theories (for example, $D = 11$ [13], type IIB [15]), and it is generally believed to hold in all theories of supergravity (although the details may vary in each particular theory depending on the specific field equations). Therefore, a necessary (but not sufficient) condition for a particular supergravity solution to preserve some supersymmetry is that the involved spacetime possesses a null or timelike Killing vector.

The existence of Killing vectors in VSI spacetimes in an arbitrary number of dimensions has been studied. It is known that there can exist no null or timelike Killing vector unless $\varepsilon = 0$ and H is independent of v (see appendix). Therefore, there are no supersymmetric solutions in any other type of supergravity theory. We therefore study only the supersymmetry properties of VSI IIB supergravity solutions with a covariantly constant null vector [16]. These are of Ricci type N and Weyl type III(a) or N [2]. We will focus on solutions of Weyl type III(a), as the Weyl type N ones have been discussed extensively in the literature. We will consider two different cases: vacuum solutions and NS-NS solutions.

The Killing spinor equation for pure gravitational solutions reads

$$\left(\partial_\mu - \frac{1}{4}w_{\mu\ ab}\Gamma^{ab}\right)\epsilon = 0. \tag{3.2}$$

Greek indices are curved indices $u, v, 1, \ldots, 8$ and Latin indices are tangent space indices $1, \ldots, 10$. The supersymmetry parameter ϵ is a complex-valued 16-component chiral spinor. In our conventions, the basis of one-forms is:

$$\begin{aligned} w^i &= dx^i \\ w^9 &= du \\ w^{10} &= dv + H\ du + W_i\ dx^i, \end{aligned}$$

and the corresponding inverse frame is:

$$\begin{aligned} e_i &= \partial_i - W_i\,\partial_v \\ e_9 &= \partial_u - H\,\partial_v \\ e_{10} &= \partial_v. \end{aligned}$$

The components of the spin connection for Weyl III(a) VSI spacetimes are

$$\begin{aligned} w_{u\,ij} &= \frac{1}{2}W_{ij}, \\ w_{u\,i9} &= -H^{(0)}{}_{,i} + W^{(0)}_{i,u} \qquad (3.3) \\ w_{i\,j9} &= \frac{1}{2}W_{ji}, \qquad (3.4) \end{aligned}$$

It has been proved[8] that the only case where supersymmetry arises is when [17]

$$\partial_k W_{ij} = 0. \qquad (3.5)$$

This condition is equivalent to the functions W_i being linear in the transverse coordinates. In that case the spacetime reduces to Weyl type N [2]. We discuss next supersymmetry on NS-NS solutions. The fermion supersymmetry transformations are given by:

$$\delta\psi_\mu = \left(\partial_\mu - \frac{1}{4}\Omega_{\mu\,ab}\Gamma^{ab}\right)\epsilon, \qquad (3.6)$$

$$\delta\lambda = \left(\not{\partial}\phi - \frac{1}{6}\not{H}\right)\epsilon, \qquad (3.7)$$

where Ω is the torsionful spin connection

$$\Omega_{\mu\,ab} = w_{\mu\,ab} + H_{\mu\,ab}. \qquad (3.8)$$

In components we have

$$\begin{aligned} \Omega_{u\,ij} &= \frac{1}{2}(W_{ij} + \tilde{B}_{ij}), \\ \Omega_{u\,i9} &= -H^{(0)}{}_{,i} + W^{(0)}_{i,u} \qquad (3.9) \\ \Omega_{i\,j9} &= \frac{1}{2}W_{ji}, \qquad (3.10) \end{aligned}$$

We consider solutions with at most one half of the supersymmetries broken;[9] i.e., $\Gamma^u\epsilon = 0$. The dilatino variation then vanishes automatically and the gravitino Killing equation reduces to

$$\left(\partial_u - \frac{1}{4}(W_{ij} + \tilde{B}_{ij})\Gamma^{ij}\right)\epsilon = 0, \quad \partial_v\epsilon = \partial_i\epsilon = 0. \qquad (3.11)$$

[8]The analysis in [17] concerns five dimensions. However, the result holds in higher dimensions so long as the transverse space is flat [18].

[9]In some cases, different fractions of supersymmetry can be preserved [20].

We see that for consistency

$$W_{ij} = \tilde{B}_{ji} + f_{ij}(u), \tag{3.12}$$

where f_{ij} are arbitrary functions of u. However, for our purposes we can take $f_{ij} = 0$ since such functions are related to Weyl type N solutions. We therefore obtain eight (complex) constant Killing spinors and half of the supersymmetry is preserved.

From eqs. (3.12) and (2.12), the metric functions W_i satisfy

$$\partial_i \left(W_{i,j} - W_{j,i}\right) = 0. \tag{3.13}$$

This is a necessary condition for the spacetime to be of Weyl type III(a). In addition it is required that

$$\partial_k \left(W_{i,j} - W_{j,i}\right) \neq 0, \tag{3.14}$$

The supersymmetry analysis is similar to that of [19] and the resulting solutions are the IIB analogues to the supersymmetric string waves. However, we have shown that the solutions can have a more general algebraic type than pp-waves. We present a few examples of such solutions below.

Consider the VSI metric[10]

$$W_1 = 0 \tag{3.15}$$

$$W_m = f_{mn}(u)x^n x^1 \tag{3.16}$$

$$H = H^{(0)}(u, x^i), \tag{3.17}$$

where f_{mn} are antisymmetric arbitrary functions of u and $m, n = 2, \ldots, 8$. This spacetime satisfies (3.13), (3.14) and is therefore of Weyl type III(a). Supported by the dilaton and Kalb-Ramond field

$$\phi = \phi(u), \quad \tilde{B}_{1m} = f_{mn}(u)x^n, \ \tilde{B}_{mn} = 2f_{nm}(u)x^1, \tag{3.18}$$

it is a supersymmetric solution of the type discussed above. The function $H^{(0)}$ can be determined from equation (2.11).

Another example involves the gyraton metric presented in [9]. In ten dimensions

$$W_i = -\frac{\tilde{p}_i(u)x^{i+1}}{Q^4} \tag{3.19}$$

$$W_{i+1} = \frac{\tilde{p}_i(u)x^i}{Q^4} \tag{3.20}$$

$$H = H^{(0)}(u, x^i), \tag{3.21}$$

where i only takes odd values $1, 3, 5, 7$ and $Q = \delta_{jk}x^j x^k$, $j, k = 1, \ldots, 8$. The $\tilde{p}_i$ are arbitrary functions [11] of u. The gyraton metric satisfies (3.13), (3.14) and so its Weyl type is III(a). In [11] this spacetime was considered in the context of supergravity, together with

[10] This metric is the ten-dimensional generalization of a five-dimensional VSI metric of Weyl type III(a) presented in [2].

[11] The relation to the functions p_i in [9] is $p_1 = \tilde{p}_1$, $p_2 = \tilde{p}_3$, $p_3 = \tilde{p}_5$, $p_4 = \tilde{p}_7$.

Ricci type	$\varepsilon = 0$	$\varepsilon = 1$
III	none	$\phi = \phi(u)$, H, F
N	$\phi = \phi(u)$, H, F	ϕ constant, H, F

Table 1: VSI supergravity solutions. H and F are given in (2.9).

a constant dilaton and Kalb-Ramond field of the form presented in the ansatz. As we have seen such solution can be generalized to include a dilaton depending on u. The (one-half) supersymmetric gyraton will be the one with

$$\phi = \phi(u), \quad \tilde{B}_{jk} = W_{kj}. \tag{3.22}$$

Again, $H^{(0)}$ can be determined from eq. (2.11). This solution belongs to the class of *saturated* string gyratons in [11]. Another supersymmetric (AdS) gyraton solution is given in [22].

4. Discussion

We have constructed solutions of IIB supergravity with NS-NS and RR fluxes and dilaton for which the spacetime has vanishing scalar invariants (VSI). The solutions are classified according to their Ricci type (N or III). The Ricci type N solutions are generalizations of pp-wave type IIB supergravity solutions. The Ricci type III solutions are characterized by a non-constant dilaton field. The resulting spacetimes are summarized in table 1. Note that we have not attempted to classify the VSI supergravity solutions in terms of their holonomy.

The supergravity solutions of Ricci type III (with $\phi = \phi(u)$) are new. We also find some new Ricci type N supergravity solutions. In addition, although the results presented above are explicitly for type IIB supergravity, similar results are expected in all supergravity theories. We have also argued that the VSI spacetimes presented are exact string solutions to all orders in α', at least in the presence of a dilaton and Kalb-Ramond field.

We have also studied the supersymmetry properties of VSI spacetimes. We have shown that VSI spacetimes whose metric functions have dependence on the light-cone coordinate v cannot posses a null or timelike Killing vector. Hence, we have argued that only VSI spacetimes with a covariantly constant null vector are candidates to preserve supersymmetry. Such spacetimes include not only pp-waves but also spacetimes of a more general algebraic type, namely, spacetimes of Weyl type III(a). We have studied the supersymmetries of vacuum and NS-NS Weyl type III(a) solutions. The latter preserve one-half of the supersymmetry when the axion and metric functions are appropriately related. We present two explicit supersymmetric examples, one of them being the string gyraton in [11]. It is likely that RR Weyl type III(a) spacetimes preserve some supersymmetry as well; we will address this question in future work.

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A. Killing vectors

In this appendix we show that there can exist no null or timelike Killing vectors in VSI spacetimes unless $\varepsilon = 0$ and H is independent of v (i.e., $\frac{\partial}{\partial v}$ is a covariantly constant null vector).

Writing the frame components of the Killing vector ξ as $\xi_{\overline{1}}\boldsymbol{n} + \xi_{\overline{2}}\boldsymbol{\ell} + \xi_{\overline{i}}\boldsymbol{m}^{\overline{i}}$, the Killing equations become:

$$\xi_{\overline{1},\overline{1}} = 0 \tag{A.1}$$

$$2\xi_{(\overline{1},\overline{2})} - \xi_{\overline{1}}H_{,v} - \xi_{\overline{i}}W_{i,v}\,\delta^{i\overline{i}} = 0 \tag{A.2}$$

$$\xi_{(\overline{1},\overline{j})} = 0 \tag{A.3}$$

$$\xi_{\overline{2},\overline{2}} + \xi_{\overline{2}}H_{,v} + \xi_{\overline{i}}[H_{,i} - W_{i,u} + HW_{i,v} - H_{,v}W_i]\delta^{i\overline{i}} = 0 \tag{A.4}$$

$$2\xi_{(\overline{2},\overline{j})} - \xi_{\overline{1}}[H_{,j} - W_{j,u} + HW_{j,v} - H_{,v}W_j] + \xi_{\overline{2}}W_{j,v} - \xi_{\overline{i}}\tilde{A}_{ij}\delta^{i\overline{i}} = 0 \tag{A.5}$$

$$\xi_{(\overline{i},\overline{j})} = 0 \tag{A.6}$$

where $\tilde{A}_{k\ell} \equiv 2W_{[k,\ell]} - 2W_{[k,|v|}W_{\ell]}$ (and is independent of v), and the directional derivatives are given by

$$\begin{aligned}
\partial_{\overline{1}} &= \partial_v \\
\partial_{\overline{2}} &= \partial_u - H\partial_v \\
\partial_{\overline{i}} &= -W_i\partial_v + \partial_{x^i}
\end{aligned} \tag{A.7}$$

(and henceforth we shall write $\partial_{x^i} = \partial_i$ for simplicity).

For the VSI metric (1.2) the functions H and W_i are given by (see (1.3) and (1.4))

$$H = \frac{1}{2}H^{(2)}v^2 + H^{(1)}v + H^{(0)}; \quad W_i = W_i^{(1)}v + W_i^{(0)}, \tag{A.8}$$

where the functions $W_i^{(0)}, H^{(0)}, H^{(1)}$ depend on (u, x^k) (i.e., are independent of v). There are two cases, $\varepsilon = 0$ and $\varepsilon = 1$, which can be represented by (see (1.5))

$$H^{(2)} = \frac{\varepsilon}{(x)^2}, \;\; x \equiv x^1; \quad W_1^{(1)} = -\frac{2\varepsilon}{x}, \;\; W_m^{(1)} = 0 \quad (m \neq 1). \tag{A.9}$$

We can immediately integrate eqs. (A.1)–(A.3) to obtain

$$\xi_{\overline{1}} = \xi(u, x^k) \tag{A.10}$$

$$\xi_{\overline{2}} = \frac{1}{2}\{\xi H^{(2)} - \xi_{,j}W_j^{(1)}\}v^2 + \{\xi H^{(1)} + L_jW_j^{(1)} - \xi_{,u}\}v + \eta(u, x^k) \tag{A.11}$$

$$\xi_{\overline{i}} = -\xi_{,i}v + L_i(u, x^k) \tag{A.12}$$

where ξ, η and L_i are arbitrary functions of (u, x^k) (and the repeated index j indicates summation). The remaining eqs. (A.4)–(A.6) become polynomials in v of order $\mathcal{O}(v^3)$, $\mathcal{O}(v^2)$, $\mathcal{O}(v)$, respectively, which can be solved to each power of v separately.

Setting the $\mathcal{O}(v)$ term in eq. (A.6) to zero, we obtain

$$\begin{aligned}\varepsilon = 0: \;\; \xi &= f_m(u)x^m + f_1(u)x + g(u)\\ \varepsilon = 1: \;\; \xi &= \frac{1}{x}f_m(u)x^m + \frac{1}{x}f_1(u) + g(u)\end{aligned} \tag{A.13}$$

where $m = 2, \ldots, N-2$ (i.e., $m \neq 1$), whence it follows that the $\mathcal{O}(v^3)$ and $\mathcal{O}(v^2)$ terms in eqs. (A.4) and (A.5), respectively, now vanish. Setting the $\mathcal{O}(v^2)$ term in eq. (A.4) to zero and the $\mathcal{O}(v)$ term in eq. (A.5) to zero, respectively, we then obtain

$$\frac{\varepsilon^2}{x^2}\left\{g' - \xi_{,u} + H^{(1)}(\xi - g) - \frac{\varepsilon}{x}L_1 + \xi_{,j}W_j^{(0)}\right\} - \xi_{,i}H^{(1)}_{,i} = 0 \tag{A.14}$$

and

$$\xi_{,k}(W^{(0)}_{k,j} - W^{(0)}_{j,k}) + 2\xi_{,j}H^{(1)} - 2\xi_{,ju} + \xi_{,k}W_k^{(0)}W_j^{(1)} + (L_k W_k{}^{(1)})_{,j} = 0. \tag{A.15}$$

A.0.1 The case $\varepsilon = 1$

Let us assume that there exists a nontrivial solution to the timelike or null Killing equations. Rotating the frame we can then align ξ_i; i.e., we can use the remaining frame freedom to choose $\xi_{\bar{i}} = \xi^I\delta^I_i$, where $\xi^I = -\xi_{,I}v + L$ and $\xi_{,J} = 0$ ($J \neq I$; i.e. $\xi = g(u, x^I)$). In this case we can then integrate eq. (A.3) to obtain $\xi_{,I} = 0$, so that $\xi = g(u)$, and consequently $L_{,i} = 0$, so that $L = \ell(u)$. Finally, eq. (A.6) implies that $\ell(u) = 0$ or $I \neq 1$, whence eq. (A.5) is satisfied identically. Hence, we have that

$$\begin{aligned}\xi_{\bar{1}} &\equiv g(u),\\ \xi_{\bar{2}} &= \frac{1}{2x^2}gv^2 + (gH^{(1)} - g')v + \eta(u, x^k),\\ \xi_{\bar{i}} &= \ell(u)\delta^I_{\bar{i}} \;\; (I \neq 1)\end{aligned} \tag{A.16}$$

The remaining eqs. to be solved become

$$\begin{aligned}\left\{\frac{1}{x^2}(\eta - gH^{(0)}) + (gH^{(1)} - g')_{,u} + \ell\left[H^{(1)}_{,I} - \frac{1}{x^2}W_I^{(0)}\right]\right\}v\\ +\{H^{(1)}\eta + \eta_{,u} - (gH^{(1)} - g')H^{(0)} + \ell[H^{(0)}_{,I} - W^{(0)}_{I,u} - H^{(1)}W_I^{(0)}]\} = 0,\end{aligned} \tag{A.17}$$

and

$$\begin{aligned}\left\{\eta_{,j} - \frac{2}{x}\eta\delta^1_j + \ell'\delta^I_j + g'W_j^{(0)} + g(W^{(0)}_{j,u} - H^{(0)}_{,j} + \frac{2}{x}\delta^1_j H^{(0)})\right.\\ \left. -\ell\left[W^{(0)}_{I,j} - W^{(0)}_{j,I} - \frac{2}{x}\delta^1_j W_I^{(0)}\right]\right\} = 0.\end{aligned} \tag{A.18}$$

Setting the $\mathcal{O}(v)$ term in eq. (A.17) to zero we obtain

$$\eta = gH^{(0)} - x^2(gH^{(1)} - g')_{,u} + \ell[W_I^{(0)} - x^2H^{(1)}_{,I}], \tag{A.19}$$

whence the remaining eqs. yield the contraints

$$2g'H^{(0)} + gH^{(0)}_{,u} - x^2\{(gH^{(1)} - g')_{,uu} + H^{(1)}(gH^{(1)} - g')_{,u}\} \\ +\ell'[W^{(0)}_I - x^2 H^{(1)}_{,I}] + \ell[H^{(0)}_{,I} - x^2\{H^{(1)}H^{(1)}_{,I} + H^{(1)}_{,Iu}\}] = 0, \quad \text{(A.20)}$$

and

$$\{g[W^{(0)}_j - x^2 H^{(1)}_{,j}]\}_{,u} + \ell'\delta^I_j + \ell\{W^{(0)}_j - x^2 H^{(1)}_{,j}\}_{,I} = 0. \quad \text{(A.21)}$$

For nontrivial functions g and ℓ, these equations reduce to constraints on the metric functions $W^{(0)}_i, H^{(1)}$ and $H^{(0)}$.

A.0.2 The case $\varepsilon = 0$

In this case, $H^{(2)} = 0, W^{(1)}_i = 0$, and

$$\xi_{\overline{1}} = f_{\hat{i}}(u)x^{\hat{i}} + f_1(u)x + g(u) = \xi(u, x^u) \\ \xi_{\overline{2}} = (\xi H^{(1)} - \xi_{,u})v + \eta(u, x^k) \\ \xi_{\overline{i}} = -f_i v + L_i$$

The remaining Killing eqs. become (summing over i)

$$\{-f_i H^{(1)}_{,i}\}v^2 + \{(\xi H^{(1)} - \xi_{,u})_{,u} + L_i H^{(1)}_{,i} - f_i[H^{(0)}_{,i} - W^{(0)}_{i,u} - H^{(1)}W^{(0)}_i]\}v \quad \text{(A.22)} \\ +\{\eta_{,u} + H^{(1)}\eta - H^{(0)}(\xi H^{(1)} - \xi_{,u}) + L_i[H^{(0)}_{,i} - W^{(0)}_{i,u} - H^{(1)}W^{(0)}_i]\} = 0,$$

and

$$\{-2f_{j,u} + 2f_i W^{(0)}_{[i,j]}\}v + \{\eta_{,j} + H^{(0)}f_j + L_{j,u} + W^{(0)}_j\xi_{,u} \\ -\xi[H^{(0)}_{,j} - W^{(0)}_{j,u}] - 2L_i W^{(0)}_{[i,j]}\} = 0. \quad \text{(A.23)}$$

The Killing vector $\frac{\partial}{\partial v}$ corresponds to the solution $f_i = L_i = 0, \xi = 0, \eta = \eta_0$ (with $H^{(1)} = 0$) in eqs. (A.22) and (A.23). Notice that there is second solution to these eqs. (corresponding to $g' = 0$) with $f_i = L_i = 0$ when $\xi = \xi_0$ and $\eta = H^{(0)}\xi_0$ (where ξ_0 is a constant), provided that $H^{(0)}_{,u} = W^{(0)}_{j,u} = 0 = H^{(1)}_{,u}$; hence this corresponds to the case in which all of the metric functions are independent of u. In this case $\xi_{\overline{1}} = \xi_0$ and $\xi_{\overline{2}} = H\xi_0$, so that $\xi = \xi_0(dv + 2Hdu + W_i dx^{i+1})$, and the corresponding Killing vector is $\xi_0\frac{\partial}{\partial u}$, as expected. Note that $|\frac{\partial}{\partial u}|^2 = 2H$, and so this Killing vector is timelike or null only when $H \leq 0$.

A.1 Timelike and null Killing vectors

By direct calculation we find that

$$|\boldsymbol{\xi}|^2 = g^{\overline{a}\overline{b}}\xi_{\overline{a}}\xi_{\overline{b}} \\ = 2\xi_{\overline{1}}\xi_{\overline{2}} + \delta^{\overline{i}\overline{j}}\xi_{\overline{i}}\xi_{\overline{j}}. \quad \text{(A.24)}$$

Assuming the Killing vector is null or timelike, we obtain

$$\left\{\left(\frac{\varepsilon\xi}{x}+\xi_{,1}\right)^2+\xi_{,\hat{i}}\xi_{,\hat{i}}\right\}v^2+2\left\{\xi\left(\xi H^{(1)}-\frac{2\varepsilon}{x}L_1-\xi_{,u}\right)-\xi_{,i}L_i\right\}v+\{2\eta\xi+L_iL_i\}\leq 0 \quad \text{(A.25)}$$

for all coordinate values in the local chart. In particular, this is satisfied for all values of v (positive and negative). Hence, it follows that

$$\left(\frac{\varepsilon\xi}{x}+\xi_{,1}\right)^2+\xi_{,\hat{i}}\xi_{,\hat{i}}=0, \quad \text{(A.26)}$$

which, since the left-hand-side is the sum of positive-definite terms, implies that

$$\frac{\varepsilon\xi}{x}+\xi_{,1}=0; \quad \xi_{,\hat{i}}=0. \quad \text{(A.27)}$$

Hence, since (A.25) is satisfied for both positive and negative values of v, we must have that

$$\xi\left(\xi H^{(1)}-\frac{2\varepsilon}{x}L_1-\xi_{,u}\right)-\xi_{,1}L_1=0, \quad \text{(A.28)}$$

and consequently

$$2\eta\xi+L_iL_i\leq 0. \quad \text{(A.29)}$$

A.1.1 The case $\varepsilon = 1$

From eq. (A.16), we have that $\xi_i=\xi\delta^I_i, \xi=g(u)$ and $L=\ell(u)$. It immediately follows from (A.27) that

$$\varepsilon\xi=0.$$

(Assuming $\varepsilon\neq 0$) we then have that $\xi=0$, and hence from eq. (A.29), $L=0$. Eq. (A.24) then implies $\eta=0$, and consequently in this case we can only obtain the trivial solution (i.e., *it there are no timelike or null Killing vectors for* $\varepsilon=1$).

A.1.2 The case $\varepsilon = 0$

From eq. (A.27) it follows that

$$\xi=g(u); \quad (f_i=0)$$

whence eq. (A.28) implies that

$$g(gH^{(1)}-g')=0$$

(and $\xi_{\overline{2}}=\eta(u,x^k), \xi_{\overline{i}}=L_i$; i.e., the components of the Killing vector have no v dependence).

If $g=0$, then (A.29) implies that $L_i=0$, whence eqs. (A.22) and (A.23) yield $\eta_{,j}=0$, so that $\eta=\eta(u)$, and $\eta'+H^{(1)}\eta=0$. In the non-trivial case ($\eta\neq 0$), this implies that $H^{(1)}=H^{(1)}(u)$. If $g=0$, we have that $gH^{(1)}-g'$, whence again $H^{(1)}=H^{(1)}(u)$. In either case, we can always effect a coordinate transformation to set $H^{(1)}=0$. In this case H has no v dependence, and *hence the spacetime admits a covariant constant null vector* $\frac{\partial}{\partial v}$ [2, 16]; that is, eqs. (A.22), (A.23) and (A.29) always admit the solution $g=0, L_j=0$, $\eta=\eta_0$ (a constant), corresponding to the null Killing vector $\frac{\partial}{\partial v}$ (and there are no further restrictions on the non-trivial metric functions $H^{(0)}, W_i^{(0)}$).

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